GOING FOR GOLD

Problem Solving

Solutions 4

Q	Topic	My Mark	Maximum Marks
1	Ratio		4
2	Probability		5
3	Polygons		3
4	Area		6
5	Pythagoras		3
6	Forming and solving equations		4
7	Percentages		4
8	Circle		5
9	Exchange rates and proportion		5
10	Volume and surface area		4
			43

Question 1 - Ratio

Peter makes a large amount of pink paint by mixing red and white paint in the ratio 2 : 3

Red paint costs £80 per 10 litres

White paint costs £15 per 10 litres.

Peter sells his pink paint in 10-litre tins for £60 per tin.

Calculate how much profit he makes for each tin he sells.

(a) Red and white paint are mixed in the ratio 2:3 to make pink paint.

   How much red and white paint are needed to make a 10 litre tin?

3 + 2 = 5

10 ÷ 5 = 2

3 x 2 = 6   2 x 2 = 4

In a 10 litre tin of pink paint, there is 4 litres of red paint and 6 litres of white paint.

(1 mark)

(b) Red paint costs £80 per 10 litres and white paint costs £15 per litres.

   How much does 1 litre of red cost? How much does 1 litre of white cost?

10 litres of red paint costs £80

1 litre of red paint costs £8

10 litres of white paint costs £15

1 litre of white paint costs £1.50

(1 mark)

(c) Use your answer to part (a) and part (c) to calculate how much it costs to make a 10 litre tin of pink paint.

4 litres of red paint costs £32

6 litres of white paint costs £9

So 10 litres of pink paint costs £32 + £9 = £41

(1 mark)

(d) Peter sells his pink paint for £60 per tin. Calculate his profit.

So Peter makes £60 - £41 = £19 profit per tin of pink paint

(1 mark)

Question 2 - Probability

There are only red counters, blue counters and green counters in a bag. There are 5 red counters. There are 6 blue counters. There is 1 green counter.
Jim takes at random a counter from the bag.

(a)   (i) Work out the probability that he takes a red counter.

There are 5 red counters and 12 counters in total.

$$\frac{5}{12}$$

(ii) Work out the probability that he takes a counter that is not red.

There are 7 counters that aren't red and 12 counters in total.

$$\frac{7}{12}$$

(3 marks)

Jim puts the counter back in the bag. He then puts some more green counters into the bag. The probability of taking at random a red counter is now $\frac{1}{3}$

(b) Work out the number of green counters that are now in the bag.

- The number of red counters in the bag has not changed. How many red counters are there in the bag?
  There is still 5 red counters in the bag.

- This number now makes up $\frac{1}{3}$ of the total counters in the bag. How many counters are in the bag?

  So
  $$\frac{1}{3} = \frac{5}{n}$$

  So the number of counters now in the bag is 15.

- How many green counters have been added into the bag?
- How many green counters are there in total?

  3 more counters have been put in the bag.
  So there is now 4 green counters in the bag.

(2 marks)
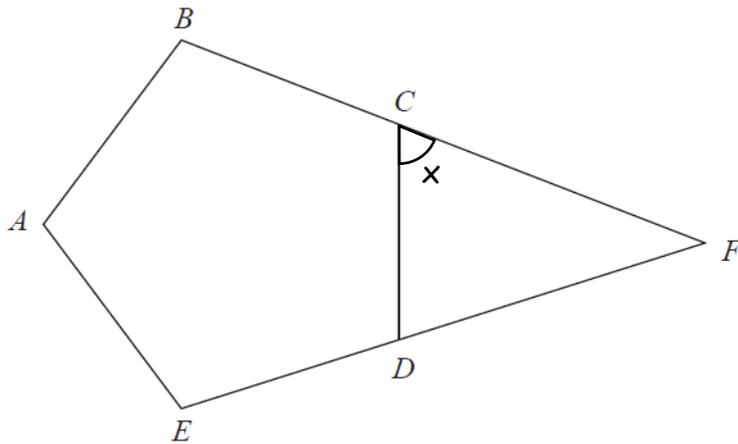
Diagram **NOT** accurately drawn

ABCDE is a regular pentagon.

BCF and EDF are straight lines.

Work out the size of angle CFD.

You must show how you got your answer.

> Information you need:
>
> For all polygons the external angles total 360
> To find the external angle of any polygon calculate
> $$360 \div n \quad \text{(where n is number of sides)}$$

a) Calculate the external angle for the pentagon (x)

The exterior angle of a pentagon = $360 \div 5 = 72^0$.

(1 mark)

b) The lines CF and DF are the same length. What type of triangle is triangle CDF? Hence find the size of angle CDF.

CDF is an isosceles triangle.

(1 mark)

c) What do the angles in a triangle add up to? Use this information to find angle CFD.

The angles in a triangle add up to 180^0.

Angle DCF = Angle FDC = 72^0.

So angle CFD = $180 - 72 - 72 = 36^0$.

(1 mark)

Question 4 - Area

Jenny is redecorating her bathroom, which is 2.8m wide, 3.5m long and 3m high.
She tiles one of the smaller walls and paints the other 3 walls.
The tiles are 35cm by 70cm and are sold in boxes of 10.
Each box costs £15.
Paint is sold in tins which costs £8.99 and covers 4.5 square metres.

Which will cost her more the paint or the tiles? How much more?

(a) Smaller walls are 280cm wide and 300cm high. What is the area of one of the smallest walls?
   Larger walls are 350cm long and 300cm high. What is the area of one of the larger walls?

Smaller walls have an area of 280 x 300 = 84000cm^2.
Larger walls have an area of 350 x 300 = 105000cm^2.

(1 mark)

(b) Tiles are 35cm by 70cm. What is the area of a tile?

Each tile has an area of 35 x 70 = 2450cm^2.

(1 mark)

(c) Using your answer to part (a) and (c), calculate how many tiles are needed to cover one of the smaller walls.

84000 ÷ 2450 = 34.28...
She needs 35 tiles.

(1 mark)

(d) Tiles come in boxes of 10. How many boxes would Jenny need to buy? Boxes cost £15 each. How much would this cost?

She will have to buy 4 boxes.
This will cost 4 x £15 = £60

(1 mark)

(e) Calculate the area of 2 larger walls and 1 smaller wall in m^2.

Area of walls to be painted = (2 x 3.5 x 3) + (2.8 x 3) = 8.4 + 21 = 29.4m^2.

(f) Each tin of paint covers 4.5 square metres. How many tins of paint would Jenny need to buy? Tins of paint cost £8.99. How much would this cost?

29.4 ÷ 4.5 = 6.53...
She will need to buy 7 tins of paint.
This will cost 7 x £8.99 = £62.93

(1 mark)

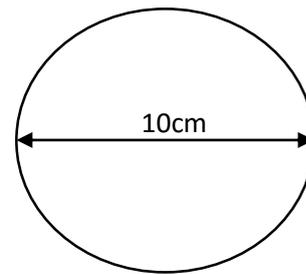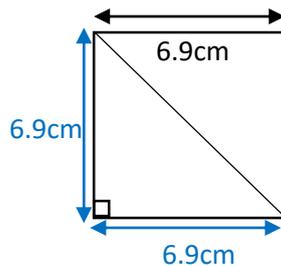(g) Using your answer to part (d) and (f): Which will cost her more the paint or the tiles? How much more?

The tiles will cost more than the paint by £2.93.

(1 mark)

Question 5 - Pythagoras

A circle has a diameter of 10cm.

A square has side lengths of 6.9cm.

6.9cm

6.9cm

6.9cm

10cm

Use Pythagoras' theorem to show that the square will fit inside the circle without touching the edge of the circle.

(a) What can you tell me about the side lengths of a square?

They are all the same length.

(b) Label the triangle above, and use Pythagoras' theorem to find the length of the diagonal.

$$a^2 + b^2 = c^2$$

$$6.9^2 + 6.9^2 = c^2$$

$$47.61 + 47.61 = c^2$$

$$95.22 = c^2$$

$$9.8 = c \text{ (1.d.p)}$$

The diagonal of the square is 9.8cm

so will fit inside the circle since its diameter is 10cm.

(2 marks)

(c) How does this show that the square will fit inside the circle?

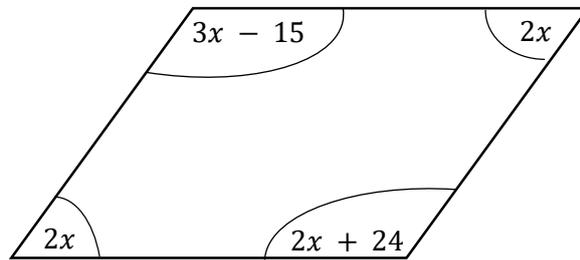The diagonal of the square is 9.8cm is its longest point.

The square will fit inside the circle since its diameter is 10cm.

(1 mark)

Question 6 - Forming and solving equations

The diagram shows a parallelogram.

Work out the sizes of the angles in the parallelogram.

(a) What do the angles in a quadrilateral add up to?

The angles in a quadrilateral add up to 360^0.

(b) Set up an equation by adding up the values of the angles in the diagram.

3x – 15 + 2x + 2x + 24 + 2x = 360

(1 mark)

(c) Simplify your equation by collecting like terms.

9x + 9 = 360

(1 mark)

(d) Solve your equation.

9x + 9 = 360
    -9   -9
9x   = 351
÷9   ÷9
 x   = 39

(1 mark)

(e) Substitute your value of x into the 4 angles to find the size of each angle.

   (Check that your answers add up to 360)

3x – 15 = 3 x 39 – 15 = 117 – 15 = 102

2x = 2 x 39 = 78

2x + 24 = 2 x 39 + 24 = 78 + 24 = 102

2x = 2 x 39 = 78

The angles are 102^0, 78^0, 102^0 and 78^0.

(1 mark)

Question 7 - Percentages

The pressure at sea level is 101325 Pascals.

Any rise of 1km above sea level decreases the pressures by 14%.

For example,

At 3km above sea level the pressure is 14% less than at 2km.

Work out the pressure at 4km above sea level.

Give your answer to 2 significant figures.

(a) What is 14% of 101325 Pascals?

14% of 101325 = 14185.5

(b) For every 1km above sea level, the pressure decreases by 14%.

What is the pressure at 1km above sea level?

1km above sea level = 101325 – 14185.5

= 87139.5 Pascals

(1 mark)

(c) Complete the table below, decreasing the pressure by 14% each time.

Use your answer to part (a) and (b) to complete the first row.

Height above sea level (km)	Pressure (Pascals)	14%	Decreased by 14%
0	101325	14185.5	87139.5
1	87139.5	12199.53	74939.97
2	74939.97	10491.5958	64448.3742
3	64448.3742	9022.772388	55425.60181
4	55425.60181		

(2 marks)

(b) What is the pressure at 4km above sea level?

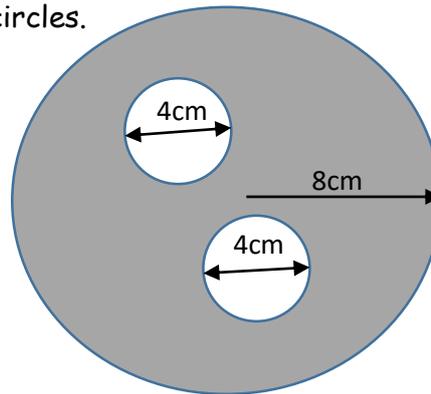At 4km above sea level, the pressure is 55000 Pascals (2.s.f)

(1 mark)

Question 8 - Circle

The diagram shows a large circle and two small circles.

The large circle has a radius of 8cm.

The two smaller circles have a diameter of 4cm.

Find the shaded area.

Information you need:

Area of a circle = π x r²   r = radius of the circle

(a) What is the area of the large outer circle?

Area of large circle = π x 8²

= 201.06cm²

(1 mark)

(b) What is the radius of the smaller circles?

Radius of the small circles = 4 ÷ 2 = 2cm

(1 mark)

(c) What is the area of one of the smaller circles?

Area of a small circle = π x 2²

= 12.57cm²

(1 mark)

(d) What is the area of the two smaller circles?

Area of the two small circles = 2 x 12.57

= 25.14cm²

(1 mark)

(e) Use your answer to part (a) and part (d) to find the shaded area.

Shaded area = 201.06 – 25.14 = 175.92cm²

(1 mark)

Question 9 - Exchange rates and proportion

Linda is going on holiday to the Czech Republic.
She needs to change some money into koruna.

She can only change her money into 100 koruna notes.

Linda only wants to change up to £200 into koruna.
She wants as many 100 koruna notes as possible.

The exchange rate is £1 = 25.82 koruna.

How many 100 koruna notes should she get?

(a) £1 = 25.82 koruna. Calculate how many koruna are in £200.

$\quad$ £1 = 25.82 koruna
£200 = 5164 koruna

(3 marks)

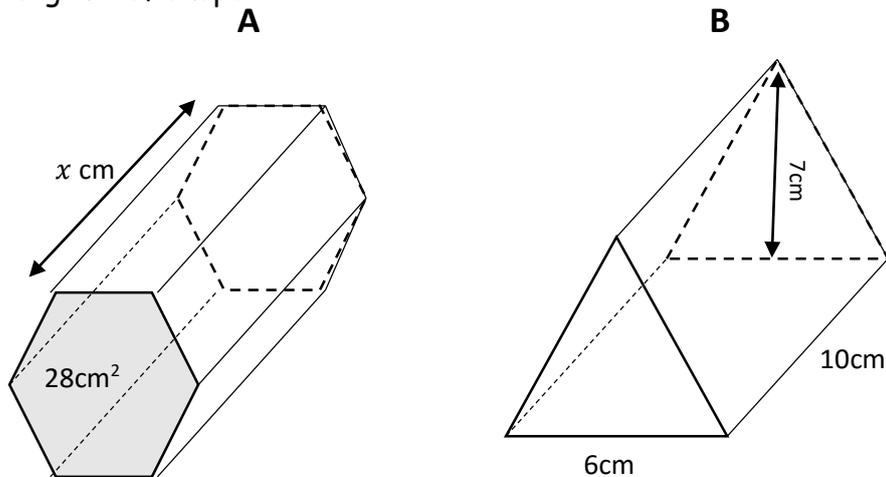(b) What is the most amount of 100 koruna notes that Linda can get?

5164 ÷ 100 = 51.64
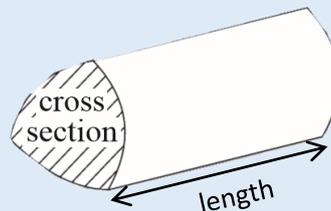
Linda should get 51 one hundred koruna notes

(2 marks)

Question 10 - Volume and surface area

The cross sectional area of shape A is 28cm².
It has a volume that is $1\frac{1}{2}$ times that of shape B.
What is the length x of shape A.

A   **B**

x cm

28cm²

7cm

10cm

6cm

Information you need:

Volume of a prism = cross sectional area x length

cross section

length

(a) Find the area of the triangular cross section of shape B.

Cross sectional area of prism B = (6 x 7) ÷ 2
                                = 42 ÷ 2
                                = 21cm²

(b) Find the volume of shape B.

Volume of prism B = 21 x 10
                  = 210cm³

(2 marks)

(c) The volume of shape A is $1\frac{1}{2}$ times as big as the volume of shape B.
    What is the volume of shape A?

Volume of prism A = 210 x 1.5
                  = 315cm³

(1 mark)

(d) What is the length of shape A?

Volume of prism A = 28 x x

28 x x = 315
    x = 315 ÷ 28
        = 11.25cm

(1 mark)