GOING FOR GOLD

Problem Solving

Solutions 3

Q	Topic	My Mark	Maximum Marks
1	Ratio		4
2	Probability		5
3	Polygons		4
4	Area		4
5	Pythagoras		5
6	Forming and solving equations		5
7	Percentages		4
8	Circle		7
9	Exchange rates and proportion		4
10	Volume and surface area		4
			46

Alison Banks - Mathematics teacher Horbury Academy
Original resource by Mary Pepper, Maths TLC

Question 1 - Ratio

Colin, Dave and Emma share some money.

Colin gets $\frac{3}{10}$ of the money.

Emma and Dave share the rest of the money in the ratio 3 : 2.

What is Dave's share of the money?

a) If Colin's share is $\frac{3}{10}$ - how much as a fraction is left for Dave and Emma?

$1 - \frac{3}{10} = \frac{7}{10}$

b) How much is left for Dave and Emma as a percentage?

$\frac{7}{10} = 70\%$

(1 mark)

c) Share the percentage for Emma and Dave in the ratio 3:2

3 + 2 = 5

70 ÷ 5 = 14

Emma gets 3 x 14 = 42

Dave gets 2 x 14 = 28

(1 mark)

d) How much is Dave's share?

28

(1 mark)

e) How much is Dave's share as a percentage?

28%

(1 mark)

Question 2 - Probability

Rhiana plays a game of bingo on the internet.

The probability that she will lose the game is 0.9.
The probability that she will win the game and share the prize is 0.05.
The probability that she will win the game and the total prize is 0.095.

Each game costs 50p.
If she wins a games she banks £3.00.
If she shares a prize she gets 2 free games.

Rhiana is going to play the game 200 times.

Work out an estimate for profit or loss that Rhiana will have made in 200 games.

a) How much does Rhiana pay to play 200 games?

200 x 50p = 10000p = £100

(1 mark)

b) Estimate how many times Rhiana will win the total prize in 200 games.

0.095 x 200 = 19

(1 mark)

c) Use part b) to find how much money Rhiana will bank form her wins

19 x 3 = £57

(1 mark)

d) Estimate how many free games Rhiana will win in 200 games.

0.05 x 200 = 10

(1 mark)

e) How many 'extra' wins can Rhiana expect in these extra games? Use your answer to adjust your answer to part c) above.

10 x 0.095 = 0.95 ≈ 1
1 x £3 = £3
£57 + £3 = £60

f) How much profit or loss can Rhiana expect to make in 200 games?

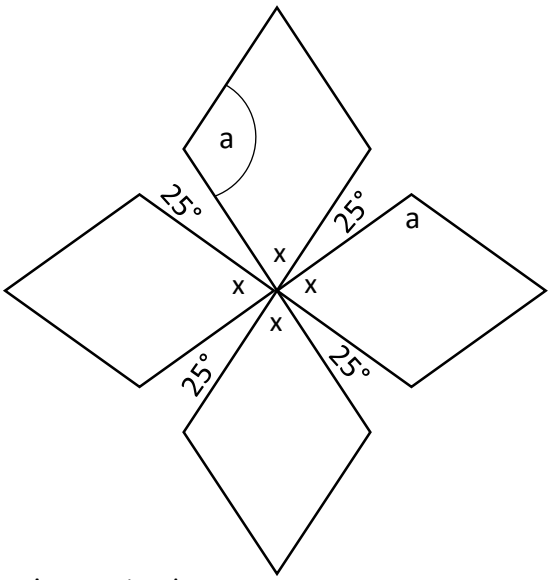
£100 - £60 = £40.

Rhiana can expect to make a £40 loss.

(1 mark)

Question 3 - Polygons

The diagram shows a pattern using four identical rhombuses.



Work out the size of angle marked a.

You must show your working

a) Explain why 4x + 100 = 360

The angles at a point add up to 360^0.

(1 mark)

b) Use part a to find the value of x

4x + 100 = 360
-100 -100
4x = 260
÷4 ÷4
x = 65

(1 mark)

c) The opposite angles in a rhombus are equal. Explain why 2x + 2a = 360

The angles in a quadrilateral add up to 360^0.

(1 mark)

d) Use part b and c above to find the value of angle a.

130 + 2a = 360
-130 -130
2a = 230
÷2 ÷2
a = 115

(1 mark)

Question 4 - Area

The diagram shows the plan of a playground.

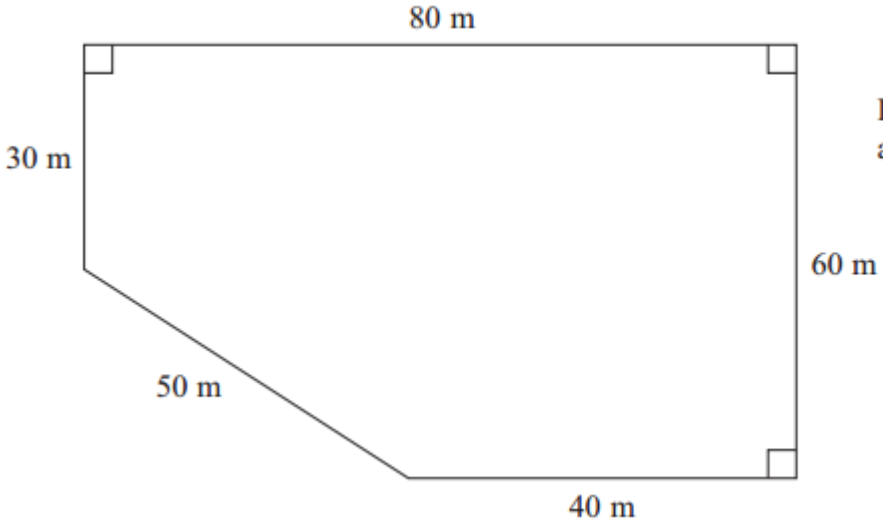


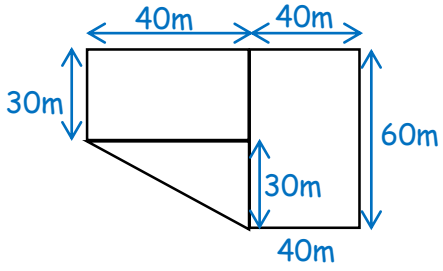
Diagram **NOT** accurately drawn

Bill is going to cover the playground with tarmac.

It costs £2.56 to cover each square metre with tarmac.

Work out the total cost of the tarmac Bill needs.

a) The playground area can be cut into two rectangles and a triangle.
Find the area of each rectangle and the area of the triangle.



Area of rectangle = 40 x 60 = 2400m^2

Area of rectangle = 40 x 30 = 1200m^2

Area of triangle = (30 x 40) ÷ 2 = 600m^2

(2 mark)

b) Use part a) to find the total area of the playground in m^2.
2400 + 1200 + 600 = 4200m^2

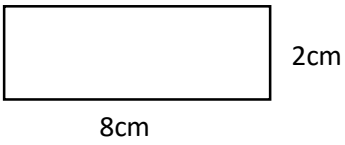
(1 mark)

c) It costs £2.56 to cover each m^2 in tarmac. Work out the total cost of covering the playground in tarmac.
4200 x 2.56 = £10752

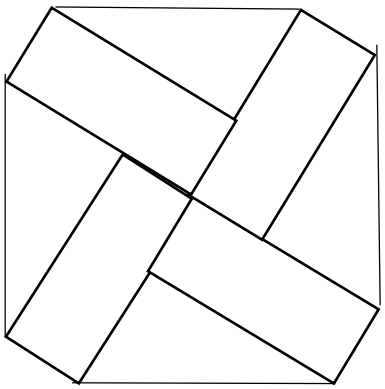
(1 mark)

Question 5 - Pythagoras

Here is a rectangle.

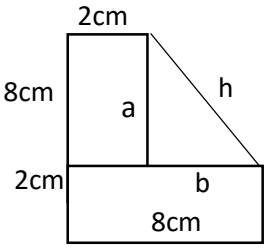


The 8 sided-shape below is made from 4 of these rectangles and 4 congruent right-angled triangles.



Work out the perimeter of the 8-sided shape.
You must show all your working.

a) The diagram is one quarter of the 8 sided shape.
What is the side lengths a and b of the right-angled triangle?
Work out the length of the hypotenuse of the triangle.

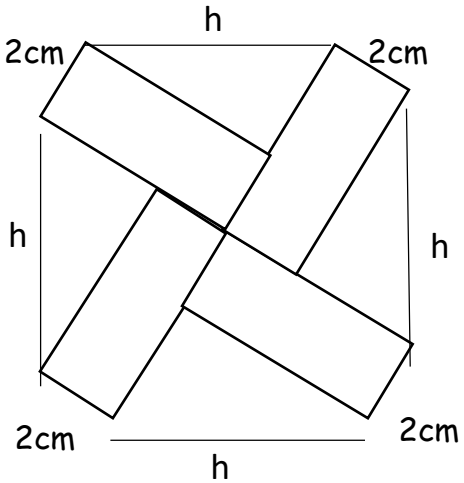


a = 8cm b = 6cm

$a^2 + b^2 = c^2$
$8^2 + 6^2 = c^2$
$64 + 36 = c^2$
$100 = c^2$
$10 = c$

(2 marks)

b) Below is the 8-sided shape



The perimeter is shown as 4 hypotenuse' and 4 lengths of 2cms.
What is the total length of the perimeter?

Perimeter = 10 + 2 + 10 + 2 + 10 + 2 + 10 + 2 = 48cm

(3 marks)

Question 6 - Forming and solving equations

The diagram shows a trapezium.

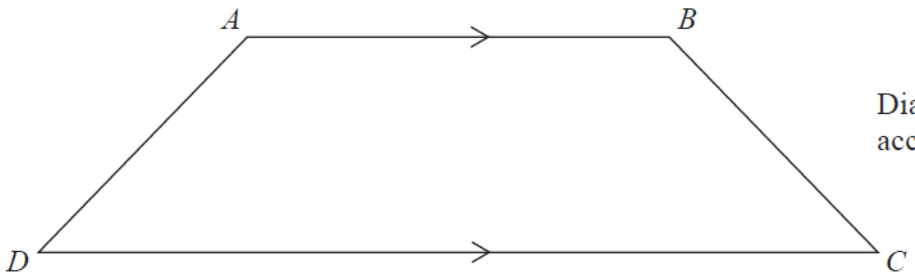


Diagram **NOT** accurately drawn

AD = *x* cm.
BC is the same length as *AD*.
AB is twice the length of *AD*.
DC is 4 cm longer than *AB*.

The perimeter of the trapezium is 38 cm.
Work out the length of *AD*.

a) AD = x and AB is twice the length. Give an expression in terms of x for the length of AB

2x

(1 mark)

b) DC is 4cm longer that AB. Use your answer to part a) to write an expression in terms of x for the length of DC.

2x + 4

(1 mark)

c) Write an expression for the perimeter of the trapezium using your answers to part a) and b) and information given in the question. Simplify the expression.

x + 2x + x + 2x + 4 = 6x + 4

(1 mark)

d) Given that the perimeter is 38cm. Using your answer to part c) write an equation and solve it for x.

6x + 4 = 38
-4 -4
6x = 34
÷6 ÷6
$x = \frac{34}{6} = \frac{17}{3}$ or $5\frac{2}{3}$

(2 marks)

Question 7 - Percentages

Railtickets and Cheaptrains are two websites selling train tickets.

Each of the websites adds a credit card charge and a booking fee to the ticket price.

Railtickets	**Cheaptrains**
Credit card charge: 2.25% of ticket price Booking fee: 80 pence	Credit card charge: 1.5% of ticket price Booking fee: £1.90

Nadia wants to buy a train ticket.

The ticket price is £60 on each website.

Nadia will pay by credit card.

Will it be cheaper for Nadia to buy the train ticket from Railtickets or from Cheaptrains?

a) To calculate the total cost of a 'Railticket' ticket, find 2.25% of £60, add it on to £60 and then add on the booking fee of 80p.

2.25% of £60 = £60 ÷ 100 x 2.25 = £1.35
Total cost of Railtickets ticket = £60 + £1.35 + £0.80 = £62.15

(1 mark)

b) Find the cost of a 'Cheaptrains' ticket including the credit card charge of 1.5% and booking fee of £1.90.

1.5% of £60 = £60 ÷ 100 x 1.5 = £0.90
Total cost of Cheaptrains ticket = £60 + £1.90 + £0.90 = £62.80

(2 marks)

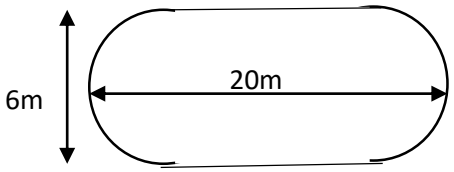
c) State which company which is the cheapest.

The cheapest company is Railtickets.

(1 mark)

Question 8 - Circle

Eliza makes this sketch of a pond.



The shortest distance across the pond is 6m.
The longest distance across the pond is 20m.

Eliza estimates that the surface area of the pond is $120m^2$.

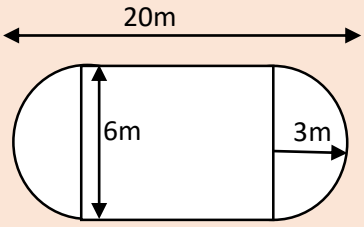
a) Explain how Eliza arrived at her estimate

She used a rectangle to estimate the area.
The area of the rectangle is $6 \times 20 = 120m^2$

(2 marks)

b) Calculate an estimate for the surface area of the pond that would be more accurate than Eliza's estimate.
Explain how you decided to calculate your estimate.
You must justify your decision.
Show all your working.

Clue to finding a more accurate estimate:
Cut the shape into a circle of radius 3m and a rectangle and then find the area of each shape as in the diagram below.



Use $\pi = 3$ to get an estimate for the area of the circle.

Area of circle = $\pi \times 3^2 \approx 3 \times 9 = 27m^2$

Area of rectangle = $6 \times (20 - 3 - 3) = 6 \times 14 = 84m^2$

Area of pond $\approx 27 + 84 = 111m^2$

(5 marks)

Question 9 - Exchange rates and proportion

All tickets for a concert are the same price.

Amy and Dan pay £63 altogether for some tickets.

Amy pays £24.50 for 7 tickets.

How many tickets does Dan buy?

a) Amy pays £24.50 for 7 tickets. What is the cost of 1 ticket?

£24.50 ÷ 7 = £3.50

(1 mark)

b) How much does Dan pay for tickets?

£63 - £24.50 = £38.50

(1 mark)

c) Use your answer to part a) and part b) to calculate the number of tickets Dan bought.

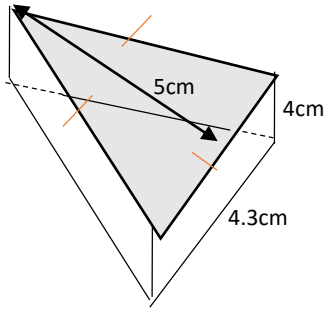
£38.50 ÷ £3.50 = 11

Dan bought 11 tickets

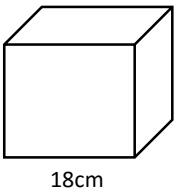
(2 marks)

Question 10 - Volume and surface area

Alex pours melted wax into moulds like this to make candles.



He melts this cube of wax.



How many candles can he make with this wax?

a) What is the area of the shaded triangle in the prism.

$(5 \times 4.3) \div 2 = 21.5 \div 2 = 10.75cm^2$

b) What is the volume of the triangular prism in cm^3?

$10.75 \times 4 = 43cm^3$

(1 mark)

c) What is the volume of the cube in cm^3?

$18 \times 18 \times 18 = 5832cm^3$

(1 mark)

d) The volume of wax (answer c)) is to be divided into trinagular candles (answer a)) How many triangular candles can be made from the wax?

$5832 \div 43 = 135.62...$

135 candles can be made from the cube.

(2 marks)