GOING FOR GOLD

Problem Solving

Solutions 2

Q	Topic	My Mark	Maximum Marks
1	Ratio		4
2	Probability		4
3	Polygons		4
4	Area		5
5	Pythagoras		4
6	Forming and solving equations		5
7	Percentages		4
8	Circle		5
9	Exchange rates and proportion		4
10	Volume and surface area		4
			43

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Original resource by Mary Pepper, Maths TLC

Question 1 - Ratio

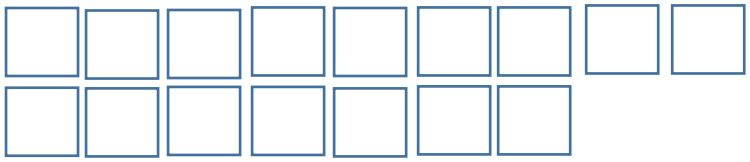
At a school

Number of boys: number of girls = 9 : 7

There are 116 **more** boys than girls.

Work out the total number of students at the school.

a) Below a diagram shows the ratio of boys to girls



If there are 116 **more** boys what does this tell you about the number in each of the last two boxes in the diagram above?

They must add up to 116.

(1 mark)

b) All the boxes in the diagram have the same number of pupils. How many boys in total are there?

Each box has 116 ÷ 2 = 58 pupils.

So there are 9 x 58 = 522 boys.

(1 mark)

c) How many girls in total are there?

There are 522 - 116 = 406 girls.

(1 mark)

d) Work out the total number of students.

There are 522 + 406 = 928 students in total.

(1 mark)

Question 2 - Probability

There are some green counters, some yellow counters, some blue counters and some red counters in a bag.

The table shows the probabilities that a counter taken at random from the bag will be green or yellow or red.

Colour	Green	Yellow	Blue	Red
Probability	0.16	0.4		0.24

There are 125 counters in the bag.

Work out the number of blue counters in the bag.

a) When you total the probability row in the table it should equal 1. Use this information to find the probability of a blue counter?

0.16 + 0.4 + 0.24 = 0.8

1 - 0.8 = 0.2

(2 marks)

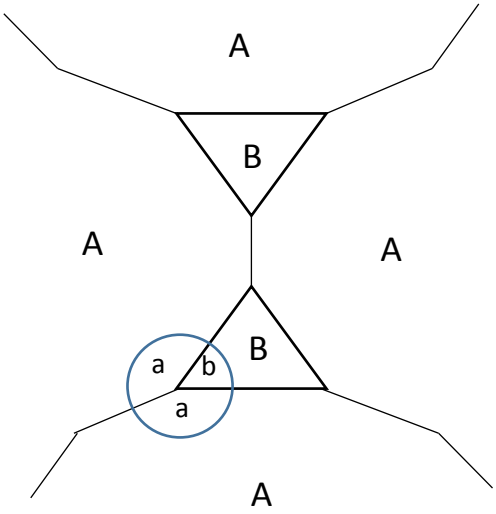
b) Use your answer to part a) to work out how many blue counters are in the bag of 125 counters?

0.2 x 125 = 25 blue counters

(2 marks)

Question 3 - Polygons

The pattern is made from two types of Tile, tile A and tile B.



Both tile A and tile B are regular polygons.

Work out the number of sides tile A has.

a) Tile B is an equilateral triangle. What is the size of the internal angle in B.
60^0.

(1 mark)

b) The angle circled is a full turn. It has two internal angles from shape A and one internal angle from shape B. Explain why 2a + b = 360.
The two angle a's and angle b meet at a point (making a full turn).

The angles at a point add up to 360^0.

So $2a + b = 360^0$.

c) Use the equation in part b, substitute angle b into the equation and then solve it to find angle a.

$2a + 60 = 360$
$-60 \quad -60$
$2a = 300$
$\div 2 \quad \div 2$
$a = 150$

(1 mark)

d) a = internal angle in shape A. Use it to find the external angle in shape A.
Internal angle + external angle = 180^0.
So external angle = $180 - 150 = 30^0$.

(1 mark)

e) 360 ÷ external angle gives the number of sides in a polygon. Use this to find the number of sides in shape A.
$360 \div$ number of sides = 30^0.
Number of sides = $360 \div 30 = 12$.

(1 mark)

Question 4 - Area

Mr Weaver's garden is in the shape of a rectangle.
In the garden

there is a patio in the shape of a rectangle
and two ponds in the shape of circles with diameter 3.8 m.

The rest of the garden is grass.

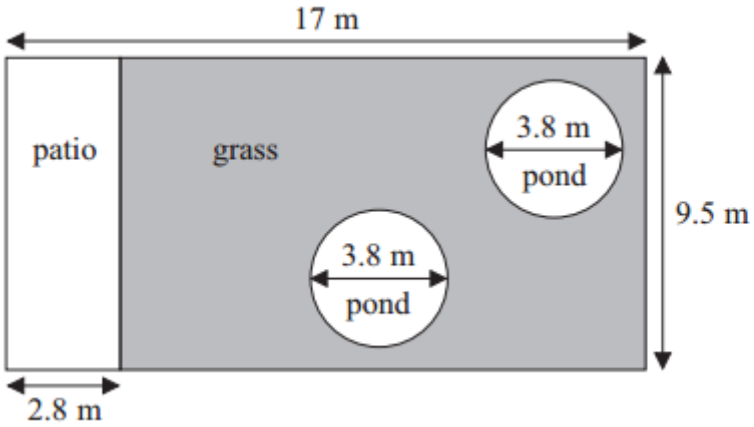


Diagram **NOT** accurately drawn

Mr Weaver is going to spread fertiliser over all the grass.
One box of fertiliser will cover 25 m^2 of grass.
How many boxes of fertiliser does Mr Weaver need?
You must show your working.

a) Work out the area of the whole garden

17 x 9.5 = 161.5m^2.

(1 mark)

b) Work out the area of the patio

2.8 x 9.5 = 26.6m^2.

(1 mark)

c) Work out the area of the ponds

Radius = 3.8 ÷ 2 = 1.9m

Area of 1 pond = π x 1.9^2 = 11.34m^2

Area of 2 ponds = x 2 = 22.68m^2

Information you need to know: Area of circle = πr^2

(1 mark)

d) Use part a) b) and c) to work out the grassed area

161.5 - 26.6 - 22.68 = 112.22m^2.

(1 mark)

e) Each box of fertiliser covers 25m^2. How many boxes of fertiliser are needed to cover the grassed area?

112.22 ÷ 25 = 4.48…

5 boxes of fertiliser are needed.

(1 mark)

Question 5 - Pythagoras

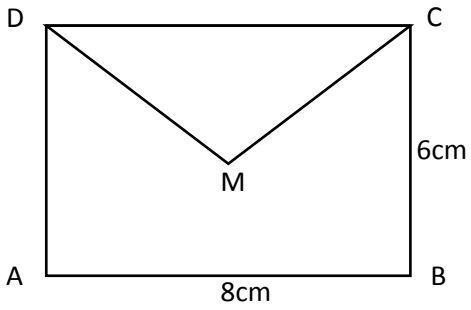
An envelope measures 6cm by 8cm.

The flap meets the envelope at the point M.

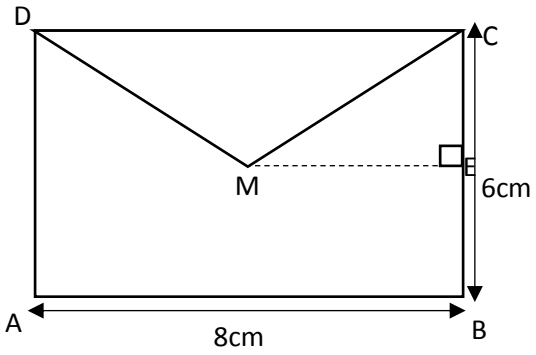
The point M is in the exact centre of the envelope.

A glue strip runs from D to M and M to C.

What is the length of the glue strip?



a) The midpoint of BC is E (see below). The triangle MCE is right angled. What are the lengths of ME and CE? How do you know?



Since M is the midpoint, the length of the line ME is 4cm and the length of the line CE is 3cm.

(1 mark)

b) Use right-angled triangle MCE to find the length of MC.

$a^2 + b^2 = c^2$

$3^2 + 4^2 = MC^2$

$9 + 16 = MC^2$

$25 = MC^2$

$5 = MC$

(2 marks)

c) MC = MD Explain why. Find the length of the glue strip from D to M and M to C.

Since M is the midpoint, MC = MD.

The length of the glue strip is 5 + 5 = 10cm

(1 mark)

Question 6 - Forming and solving equations

The diagram shows a square.

All the lengths are measured in centimetres.

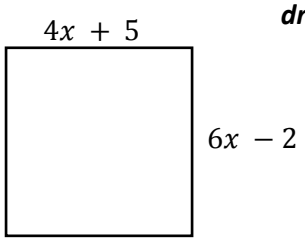


Diagram not drawn to scale

Use an algebraic method to find the length of one side of the square.

a) The shape is a square. Explain why 4x + 5 = 6x – 2

The side length of a square are the same.

(1 mark)

b) Solve the equation in a) to find the value of x

4x + 5 = 6x - 2

-4x -4x

5 = 2x - 2

+2 +2

7 = 2x

÷2 ÷2

3.5 = x

(2 marks)

c) Substitute the value for x (from part b) above) into one of the given sides to find the length of side of the square.

Side length of square = 4x + 5

= 4 x 3.5 + 5

= 14 + 5

= 19cm

(2 marks)

Question 7 - Percentages

In a company, the ratio of the number of men to the number of women is 3 : 2

40% of the men are under the age of 25

10% of the women are under the age of 25

What percentage of all the people in the company are under the age of 25?

a) If the company has 100 people and the ratio of men to women is 3:2. How many of the people are men and how many of them are women?

3 + 2 = 5

100 ÷ 5 = 20

There would be 3 x 20 = 60 men

There would be 2 x 20 = 40 women

(1 mark)

b) 40% of the men are under 25. How many men are under 25?

10% of 60 = 6

40% of 60 = 24

There are 24 men under 25.

(1 mark)

c) 10% of the women are under the age of 25. How many women are under the age of 25?

10% of 40 = 4

There are 4 women under the age of 25.

(1 mark)

d) Use your answer to part c) and part d) to find the total number of people under the age of 25. What percentage of the people are under the age of 25?

There are 24 + 4 = 28 people under the age of 25.

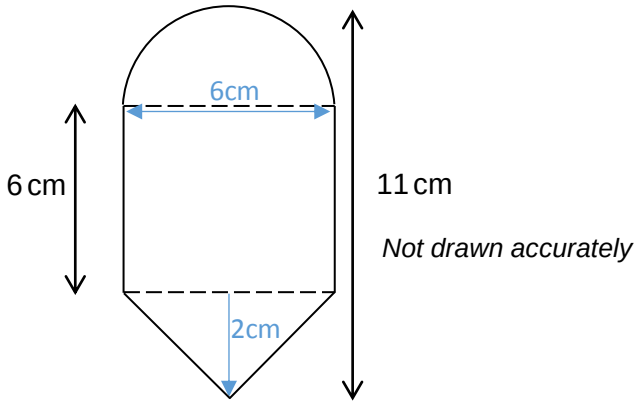
28% of people are under the age of 25.

(1 mark)

Question 8 - Circle

A badge is made from an isosceles triangle, a square and a semi-circle.

Work out the area of the badge.



a) Find the area of the square

Area of square = 6 x 6 = 36cm^2

(1 mark)

b) Find the area of the circle

Radius = 6 ÷ 2 = 3cm

Area of circle = π x 3^2 = 28.3cm^2

(2 marks)

c) Find the area of the triangle

Area of triangle = (6 x 2) ÷ 2 = 6cm^2

(1 mark)

d) What is the total area of the badge?

Total area of badge = 36 + (28.3 ÷ 2) + 6

= 36 + 14.15 + 6

= 56.15cm^2

(1 mark)

Question 9 - Exchange rates and proportion

Here is a list of ingredients for making 18 mince pies.

Ingredients for 18 mince pies
225 g of butter
350 g of flour
100 g of sugar
280 g of mincemeat
1 egg

Elaine wants to make 45 mince pies.

Elaine has

1 kg of butter
1 kg of flour
500 g of sugar
600 g of mincemeat
6 eggs

Does Elaine have enough of each ingredient to make 45 mince pies?
You must show clearly how you got your answer.

Clue: 18 + 18 + 9 = 45

Flour required for 45 pies: 225g + 225g + (1/2 of 225)g = 562.5g flour (Elaine has enough flour)

Complete a similar calculation for each of the other ingredients and then check what Elaine has.

45 ÷ 18 = 2.5

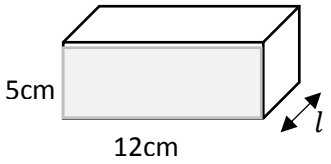
Ingredients for 45 mince pies:	Elaine has:		Does she have enough?
562.5g of butter	1	kg of butter	Yes
875g of flour	1	kg of flour	Yes
250g of sugar	500	g of sugar	Yes
700g of mincemeat	600	g of mincemeat	No
2.5 eggs	6	eggs	Yes

No, Elaine does not have enough. She would need to buy 100g of mincemeat.

(4 marks)

Question 10 - Volume and surface area

This cuboid has a volume of $360cm^3$.



Work out its surface area.

a) Work out the area of the shaded face of the cuboid.

$5 \times 12 = 60cm^2$

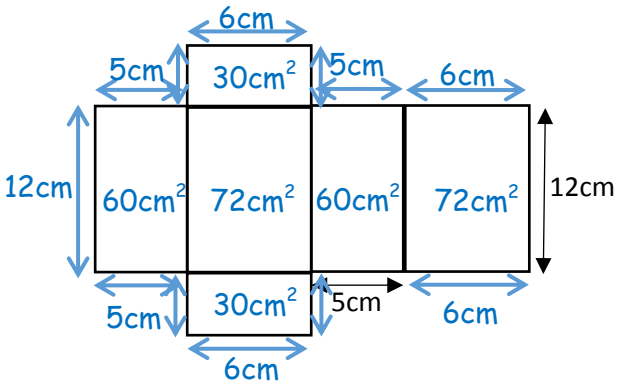
b) The volume of a prism is cross-sectional area x length. The volume is $360cm^3$. Use this to work out the length of the cuboid.

$60 \times \text{length} = 360$

$\text{Length} = 360 \div 60 = 6cm$

(1 mark)

c) The net of the cuboid is sketched below



Fill in any further detail (use your answer to part b)) and then use the net to help you find the surafce area of the cuboid.

$\text{Surface area} = 60 + 30 + 72 + 30 + 60 + 72$

$= 324cm^2$

(3 marks)