GOING FOR GOLD

Problem Solving

Solutions 1

Q	Topic	My Mark	Maximum Marks
1	Ratio		4
2	Probability		5
3	Polygons		4
4	Area		4
5	Pythagoras		5
6	Forming and solving equations		5
7	Percentages		5
8	Circle		4
9	Exchange rates and proportion		4
10	Volume and surface area		6
			46

Question 1 - Ratio

White paint cost £2.80 a litre.
Blue paint cost £3.50 a litre.
White and blue paint are mixed in the ratio 3:2

Work out the cost of 18 litres of the mixture.

a) To make 5 litres of paint I would put 3 litres of white and 2 litres of blue paint. How much would 5 litres of the mixture cost?

3 × £2.80 = £8.40

2 × £3.50 = £7.00

5 litres of paint = £15.40

(2 marks)

b) How much would 1 litre of the mixture cost?

1 litre = £15.40 ÷ 5 = £3.08

(1 mark)

c) Work out the cost of 18 litres of the mixture.

18 litres = 18 × £3.08 = £55.44

(1 mark)

Question 2 - Probability

Karl wants to raise money for charity. He designs a game for people to play.

Karl uses a ten sided dice for the game.   The dice is numbered 1 to 10.

Each person will roll the dice once. A person wins the game if the dice lands on a multiple of 4.

Ali plays the game once.

    a) Work out the probability that Ali will win the game.

        i.   How many multiples of 4 are in the numbers 1 to 10?

              4 and 8. There are 2 multiples of 4.

        ii.   What is the probability of getting a multiple of 4 if you choose one number from 1 to 10?

$$^2/_{10} = ^1/_5$$

<div align="right">(2 marks)</div>

Each person pays 30p to play the game once. The prize for a win is £1.

Karl thinks the game will be played 100 times.

    b) Work out an estimate for how much money Karl will raise for charity.

        i.   If the game is played 100 times and each person pays 30p to play. How much money will the charity collect from games (income)?

              100 x 30 = 3000p = £30

        ii.   If the game is played 100 times how many times will there be a winner (use part a to find the answer)

$$^1/_5 \text{ of } 100 = 20 \text{ times}$$

        iii.   How much money will be paid in winnings (outgoings)?

              20 x £1 = £20

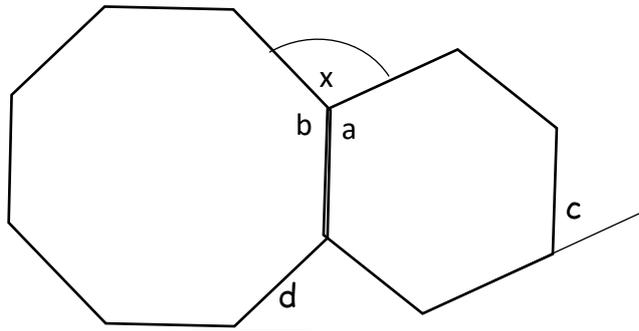   iv.   How much profit will be made (Income – outgoings)?

              £30 - £20 = £10

<div align="right">(3 marks)</div>

Question 3 - Polygons

The diagram shows a regular hexagon and a regular octagon.

Find angle x.

> Information you need:
>
> For all polygons the external angles total 360
> To find the external angle of any polygon calculate
> $$360 \div n \quad \text{(where n is number of sides)}$$

a) Calculate the external angle for the hexagon (c) and the octagon (d).

> Information you need:
> The internal angle for any polygon is: 180 – external angle

External angle of hexagon = $360 \div 6 = 60^0$

External angle of octagon = $360 \div 8 = 45^0$

(1 mark)

b) Calculate the internal angle for the hexagon (a) and the octagon (b).

Internal angle of hexagon = $180 - 60 = 120^0$

Internal angle of octagon = $180 - 45 = 135^0$

(1 mark)

c) On the diagram label angle a and angle b. Use this information to find angle x.

Angle x = $360 - 135 - 120 = 360 - 255 = 105^0$

(2 marks)

Question 4 - Area

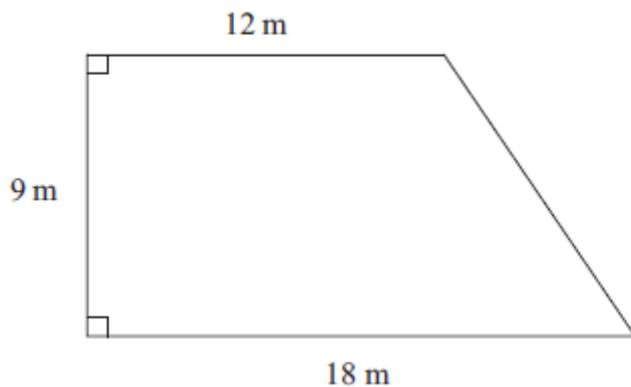Here is a diagram of Jim's garden.

12 m

Diagram **NOT**
accurately drawn

9 m

18 m

Jim wants to cover his garden with grass seed to make a lawn.
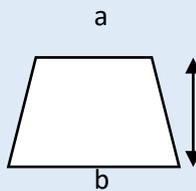
Grass seed is sold in bags.

There is enough grass seed in each bag to cover 20 m² of garden.

Each bag of grass seed costs £4.99

Work out the least cost of putting grass seed on Jim's garden.

> a
>
> Information you need to know:
>
> Area of a trapezium = ½ (a+ b) x h
>
> b

a) Calculate the area of the trapezium

Area of trapezium   = ½ (12 x 18) x 9

                   = ½ x 30 x 9

                   = 135m²

(2 marks)

b) Find the number of bags of grass seed required to cover the area of the garden (each bag covers 20m²)?

20 x 6 = 120m²
20 x 7 = 140m² <- He would need 7 bags

(1 mark)

c) Each bag of grass seed costs £4.99. Find the cost of the grass seed.

7 x £4.99 = £34.93

(1 mark)

ABCD is a trapezium.
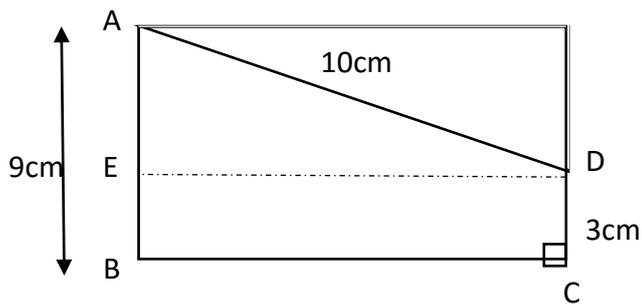
AD = 10 cm
AB = 9 cm
DC = 3 cm
Angle ABC = angle BCD = 90°

Calculate the length of AC.
Give your answer correct to 3 significant figures.

a) Use the triangle ADE to find the length of DE (note AE = AB – CD)

$a^2 + b^2 = c^2$

$DE^2 + 6^2 = 10^2$

$DE^2 + 36 = 100$

$DE^2 = 64$

$DE = 8cm$

(2 marks)

b) Use the answer to part a) and the triangle ABC to find the length of AC

$a^2 + b^2 = c^2$

$9^2 + 8^2 = c^2$

$81 + 64 = c^2$

$145 = c^2$

$c = 12.0cm$ (3 sig fig)

(3 marks)

Question 6 - Forming and solving equations

ABC is a triangle.

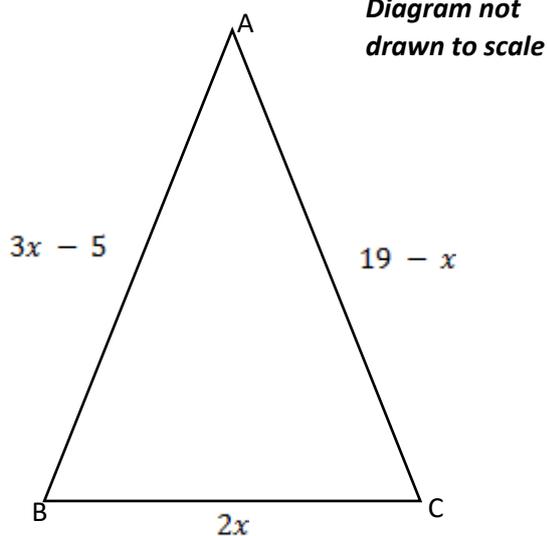*Diagram not drawn to scale*

Angle ABC = angle BCA

The length of AB = $(3x - 5)$ cm

The length of AC = $(19 - x)$ cm

The length of BC = $2x$ cm

Work out the perimeter of the triangle.

Give your answer as number of centimetres.

A

$3x - 5$

$19 - x$

B $2x$ C

a) Angle ABC = Angle BCA what type of triangle is it?

   Isosceles

(1 mark)

b) This means that AB = AC. Write an equation and solve it.

   3x - 5 = 19 - x
   +x   +x
   4x - 5 = 19
       +5   +5
   4x   = 24
   ÷4   ÷4
    x   = 6

(2 marks)

c) Use your answer to part b) to give the lengths of the sides of the triangle.

   AB = 3x - 5 = 18 - 5 = 13cm
   AC = 19 - x = 19 - 6 = 13cm
   BC = 2x = 12cm

(1 mark)

d) Use part c) to give the perimeter of the triangle.

   13 + 13 + 12 = 38cm

(1 mark)

Question 7 - Percentages

Mr Watkins needs to buy some oil for his central heating.

Mr Watkins can put up to 1500 litres of oil in his oil tank.

There are already 850 litres of oil in the tank.

Mr Watkins is going to fill the tank with oil.

The price of oil is 67.2p per litre.

Mr Watkins gets 5% off the price of the oil.

How much does Mr Watkins pay for the oil he needs to buy?

a) Find how many litres of oil is required to fill the oil tank.

1500 - 850 = 650 litres

(1 mark)

b) Oil costs 67.2p per litre. What is the cost of the oil?

67.2 x 650 = 43680p = £436.80

(2 marks)

c) Mr Watkins gets 5% off the price of the oil. Calculate the price he will pay for the oil.

10% of £436.80 = £43.68

5% of £436.80 = £21.84

Price Mr Watkins pays = £436.80 - £21.84 = £414.96

(2 marks)

Question 8 - Circle

The diagram shows a circle drawn inside a square.

The circle has a radius of 6 cm.
The square has a side of length 12 cm.
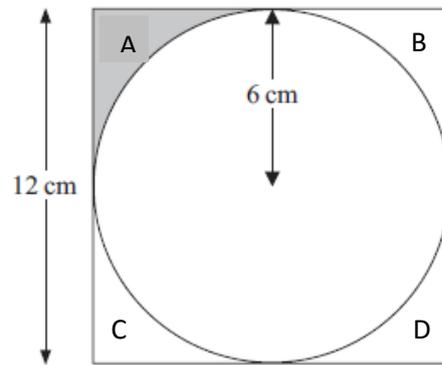
Work out the shaded area.
Give your answer in terms of π.

Diagram **NOT** accurately drawn

a)   What is the area of the square?

Area of square = 12 x 12 = 144cm²

(1 mark)

b)   What is the area of the circle?

Area of circle = πr^2 = π x 6 x 6 = 36π

(1 mark)

c)   By subtracting area in b) from the area a), find the area of the total of A, B , C and D

Area of all corner pieces = 144 - 36π

(1 mark)

d) The shaded area is ¼ of the answer in part c). Find the shaded area.

Shaded area = 36 - 9π

(1 mark)

Question 9 - Exchange rates and proportion

In the UK, petrol costs £1.24 per litre.
In the USA, petrol costs 3.15 dollars per US gallon.

1 US gallon = 3.79 litres
£1 = 1.47 dollars

Is petrol cheaper in the UK or in the USA?

a)   1 US gallon = $3.15
        3.79 litres = 1 US gallon
How many litres of petrol can you buy for $3.15 in the USA?

   Petrol costs $3.15 per US gallon.
   1 US gallon = 3.79 litres.
   So 3.79 litres will also cost $3.15.

                                                            (1 mark)

b) Use your answer to part a) to find out how much it costs for 1 litre of petrol in USA
     dollars.

   1 litre costs $3.15 ÷ 3.79 = $0.83

                                                            (1 mark)

c) Use your answer to part b) and the exchange rate £1 = 1.47 dollars to find the cost per
     litre of petrol in the USA to cost in UK £'s.

        £1 = $1.47
     £0.56 = $0.83

                                                            (1 mark)

d)   Compare your answer to c) with the cost of petrol in the UK to state which country is
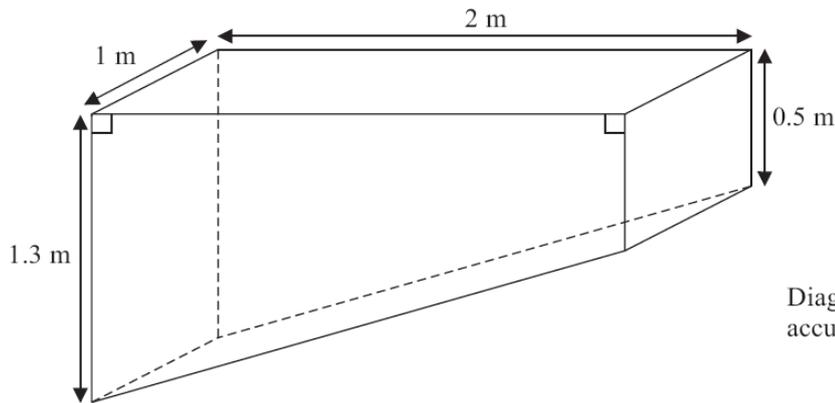   cheaper per litre of petrol.

   Petrol costs £1.24 in the UK.
   Petrol costs £0.56 in the USA.

   Petrol is cheaper in the USA.

                                                            (1 mark)

Question 10 - Volume and surface area

Sumeet has a pond in the shape of a prism.

Diagram **NOT** accurately drawn

The pond is completely full of water.
Sumeet wants to empty the pond so he can clean it.
Sumeet uses a pump to empty the pond.

The volume of water in the pond decreases at a constant rate.
The level of the water in the pond goes down by 20 cm in the first 30 minutes.

Work out how much more time Sumeet has to wait for the pump to empty the pond completely.

a) The cross-sectional area of the pond is a trapezium. What is the area of the trapezium?

$\frac{1}{2}$ (1.3 + 0.5) × 2 = $\frac{1}{2}$ × 1.8 × 2 = 1.8m²

(1 mark)

b) What is the volume of the prism?

1.8 × 1 = 1.8m³

(1 mark)

c) What is the volume of water that is pumped out of the pond in the first 30 minutes? (Note: it is a cuboid measuring 0.2m by 2m by 1m)

0.2 × 2 × 1 = 0.4m³

(2 mark)

d) Use part c) to find the volume of water pumped per hour.

0.4 × 2 = 0.8m³ per hour

e) Use your answer to part b) and part d) to find the time taken to empty the pond.

1.8 ÷ 0.8 = 2.25 = 2 hours 15 minutes

(1 mark)

f) Use the answer to part e) to state the additional time to empty the pond after the first 30 minutes.

2 hours 15 minutes - 30 minutes = 1 hour 45 minutes

(1 mark)