A* A B C D

Completing the square

If a quadratic expression is written in the form $(x + p)^2 + q$, it is in COMPLETED SQUARE FORM.

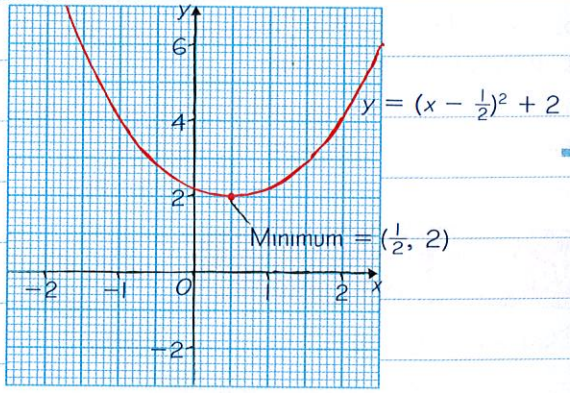
Completing the square

You can use this formula to complete the square:

$x^2 + 2bx + c = (x + b)^2 - b^2 + c$

Minimum values

Quadratic graphs are curves. This quadratic graph has a minimum point. You can find the coordinates of the minimum point by completing the square.



The minimum point of the graph $y = (x + p)^2 + q$ is at $(-p, q)$.

Learn this if you're going for an A*.

Worked example

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(a) Write $x^2 + 6x + 20$ in the form $(x + p)^2 + q$

$x^2 + 6x + 20 = (x + 3)^2 - 3^2 + 20$
$= (x + 3)^2 - 9 + 20$
$= (x + 3)^2 + 11$

(b) What is the minimum value of $x^2 + 6x + 20$?

Minimum value = 11

(a) Look at the formula for completing the square.
$2b = 6$ so $b = 3$ and $c = 20$
Substitute these values into the formula.

(b) $(x + 3)^2$ is a square, so it can't be negative. The minimum value for $(x + 3)^2$ is 0, so the minimum value for $(x + 3)^2 + 11$ is 11.

Check it!
Multiply out the brackets:
$(x + 3)^2 + 11 = x^2 + 3x + 3x + 9 + 11$
$= x^2 + 6x + 20$ ✓

Solving the equation

Once you have completed the square, the unknown only appears once in the equation. You can solve the equation using inverse operations.

$x^2 - 8x + 1 = 0$
$(x - 4)^2 - 16 + 1 = 0$
$(x - 4)^2 - 15 = 0$ (+ 15)
$(x - 4)^2 = 15$ (√)
$x - 4 = \pm\sqrt{15}$ (+ 4)
$x = 4 \pm \sqrt{15}$

When you square root both sides, you need to add a ± sign in front of the square root.

Completing the square is a useful method for solving equations when you need to give your answer as a SURD.

Now try this

Divide each term by 2 before you start.

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1. Show that $x^2 - 4x + 15$ can be written as $(x + p)^2 + q$ for all values of x.
State the values of p and q. **(3 marks)**

2. Solve $2x^2 + 8x - 3 = 0$
Give your answers in the form $p \pm \sqrt{\frac{q}{r}}$ where p, q and r are integers. **(4 marks)**

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